

MSSM FROM SUSY TRINIFICATION

G. Lazarides and C. Panagiotakopoulos

Physics Division
School of Technology
University of Thessaloniki
Thessaloniki, Greece

Abstract

We construct a $SU(3)^3$ supersymmetric gauge theory with a common gauge coupling g using only fields belonging to 27 , $\overline{27}$ and singlet representations of E_6 . Spontaneous breaking of this gauge group at a scale $M_X = 1.3 \times 10^{16} \text{GeV}$ gives naturally rise exactly to the Minimal Supersymmetric Standard Model (*MSSM*) and consequently to the experimentally favored values of $\sin^2 \theta_w$ and α_s . The gauge hierarchy problem is naturally solved by a missing-partner-type mechanism which works to all orders in the superpotential. The baryon asymmetry can be generated in spite of the (essential) stability of the proton. The solar neutrino puzzle is solved by the MSW mechanism. The LSP is a natural "cold" dark matter candidate and "hot" dark matter might consist of τ -neutrinos. This model could be thought of as an effective $4d$ theory emerging from a more fundamental theory at a scale $M_c = M_P / \sqrt{8\pi}$ where $a_G \equiv \frac{g^2}{4\pi}$ happens to be equal to unity.

The Minimal Supersymmetric Standard Model (MSSM) with a supersymmetry breaking scale $M_s \sim 1TeV$ is remarkably consistent with unification of the three gauge coupling constants at a scale $\sim 10^{16}GeV$ assuming the observed values for $\sin^2\theta_w$ and α_s at the electroweak scale. This remarkable fact has been widely advertized ⁽¹⁾ as evidence for supersymmetry in connection with the minimal supersymmetric $SU(5)$ model⁽²⁾ which predicts exactly the spectrum of the *MSSM* below a unification scale $M_X \sim 10^{16}GeV$.

Despite this undoubted success, however, it appears that the minimal $SU(5)$ scheme faces several difficulties. For instance, the predicted asymptotic mass relations involving the first two families seem to be in contradiction with the observations. Furthermore, the close relationship between the light fermion masses and proton decay through $d = 5$ operators together with the lack of observation of such events could soon lead to exclusion of the minimal scheme. Additional reasons for abandoning the minimal $SU(5)$ framework come from considerations of neutrino masses, baryon asymmetry, etc. Finally, one should not forget the gauge hierarchy problem.

In a recent paper⁽³⁾ an effort has been made to overcome some of the difficulties of the minimal supersymmetric $SU(5)$ model by embedding the standard group $SU(3)_c \times SU(2)_L \times U(1)_Y$ in $G \equiv SU(3)_c \times SU(3)_L \times SU(3)_R$. This gauge group, which often arises in the simplest compactification schemes of the heterotic $E_8 \times E_8$ superstring theory, has several advantages. Leptons and quarks belong to separate representations which results in avoiding wrong mass relations between light fermions. For the same reason, one avoids the close relationship between the light quark masses and the proton decay amplitude through $d = 5$ interactions encountered in minimal unifica-

tion schemes like $SU(5)$ or $SO(10)$. The imposition of a simple Z_2 symmetry proved sufficient to guarantee the almost complete absence of baryon number violation in the perturbative sector of the theory. The observed baryon asymmetry was then generated by combining a leptogenesis mechanism with the non-perturbative baryon-and-lepton-number-violating effects of the standard electroweak theory. In this first attempt specific models were constructed which were "close" to the MSSM below a unification scale $M_X \sim 10^{16} GeV$, but the gauge hierarchy problem was not addressed at all.

The purpose of the present note is to construct a specific model based on the gauge group $G \equiv SU(3)_c \times SU(3)_L \times SU(3)_R$ which below a unification scale $M_X \sim 10^{16}$ GeV gives naturally rise exactly to the MSSM. This guarantees the successful "predictions" of the MSSM for $\sin^2\theta_w$ and α_s providing, in addition, a reason for the asymptotic equality of the three gauge coupling constants. We assume that the theory emerges as a low energy effective theory from a more involved construction at a scale M_c close to the Planck mass $M_P \simeq 1.2 \times 10^{19} GeV$. At the scale M_c , the three $SU(3)$ gauge couplings are equal and, due to a symmetric spectrum among them, they remain equal (in the one loop approximation) until the scale $M_X \sim 10^{16} GeV$ where G breaks down to the standard gauge group. Using appropriate discrete symmetries we forbid (essentially completely) proton decay and we succeed in solving the gauge hierarchy problem. Actually, we obtain only one light pair of electroweak higgs doublets which does not acquire any mass from renormalizable as well as non-renormalizable superpotential terms as long as an anomalous Z_2 discrete symmetry remains unbroken. Our construction makes use only of fields contained in the 27-dimensional and singlet representations of E_6 .

All superpotential couplings are assumed to be generically of order unity. The mass spectrum entering the renormalization group (RG) equations for $\sin^2\theta_w$ and α_s is completely determined by quadratic and cubic superpotential terms only. This is achieved by an appropriate (minimal) choice of the field content. Therefore, M_c does not enter the calculation for $\sin^2\theta_w$ and α_s but is only determined as the scale where the unified G gauge structure constant α_G becomes equal to unity.

The left handed lepton, quark and antiquark superfields transform under G as follows:

$$\begin{aligned}\lambda &= (1, \bar{3}, 3) = \begin{pmatrix} H^{(1)} & H^{(2)} & L \\ E^c & \nu^c & N \end{pmatrix}, \\ Q &= (3, 3, 1) = \begin{pmatrix} q \\ g \end{pmatrix}, \\ Q^c &= (\bar{3}, 1, \bar{3}) = \begin{pmatrix} u^c, d^c, g^c \end{pmatrix}.\end{aligned}$$

Here N and ν^c denote standard model singlet superfields while $g(g^c)$ is an additional down-type quark (antiquark). We will be working with eight fields of each type (λ, Q, Q^c) and five corresponding mirror fields $(\bar{\lambda}, \bar{Q}, \bar{Q}^c)$. Notice that the field content is chosen to ensure identical running of the three $SU(3)$ gauge couplings in the G-symmetric phase.

We impose invariance under three discrete Z_2 symmetries P, C and S . Under P all $\lambda, \bar{\lambda}$ fields remain invariant while all $Q, \bar{Q}, Q^c, \bar{Q}^c$ fields change sign. This has the effect of forbidding all cubic superpotential couplings involving three quark or three antiquark fields. This fact combined with the lack of gauge boson mediated proton decay leads to a practically stable proton. Under C all fields remain invariant except $\lambda_5, \lambda_8, \bar{\lambda}_3$ and $\bar{\lambda}_5$ which change sign. Finally, under S all fields remain invariant, except $\lambda_5, \lambda_6, \lambda_7, \bar{\lambda}_3, \bar{\lambda}_4$,

Q_1, Q_2 and Q_3^c which change sign.

The symmetry breaking of G down to the standard group is obtained at the scale $M_X \sim 10^{16} GeV$ through appropriate superpotential couplings of the fields acquiring a superlarge vacuum expectation value. These are the fields $\lambda_7, \bar{\lambda}_4, \lambda_8, \bar{\lambda}_5$. The superlarge expectation values are: $\langle N_7 \rangle = \langle \bar{N}_4 \rangle^* = \langle \nu_8^c \rangle = \langle \bar{\nu}_5^c \rangle^*$. We also introduce in the superpotential explicit mass terms of order M_X for all Q, \bar{Q} and Q^c, \bar{Q}^c pairs as well as all $\lambda, \bar{\lambda}$ pairs involving fields which do not acquire vacuum expectation values.

All vacuum expectation values leave the discrete symmetry P unbroken. The discrete symmetry C combined with the Z_2 discrete subgroup generated by the element $(diag(-1, -1), diag(-1, -1))$ of $SU(2)_L \times SU(2)_R$ gives a Z_2 group C' which remains unbroken by all the vacuum expectation values and acts as "matter parity". Finally the discrete symmetry S combined with the discrete subgroup generated by the element $diag(-1, +1, -1)$ of $SU(3)_R$ gives a Z_2 group S' which remains unbroken by the superlarge expectation values. The role of P in suppressing proton decay has been explained earlier. The "matter parity" C' plays an essential role in suppressing some lepton number violating couplings at the level of the MSSM. This is important for the implementation of our mechanism for generating the baryon asymmetry of the universe as well as in stabilizing the lowest supersymmetric particle (LSP). Finally the combined effect of C' and S' solves in a natural manner the gauge hierarchy problem.

We now turn to a brief discussion of the fermion mass spectrum below the scale M_X . [The bosonic mass spectrum is directly obtainable from the fermionic one, since supersymmetry is assumed to be exact for energies much

above $M_s \sim 1TeV$.] A consequence of the fact that the Z_2 symmetries C' and S' remain unbroken by the superlarge vacuum expectation values is that all states can be classified by their C' and S' charge. The mass matrices of the various sectors are, therefore, broken up into submatrices involving only states characterized by common values of C' and S' . The symmetry P leaves all mass terms invariant and therefore could not have any consequences as far as the mass spectrum is concerned.

We will only describe in some detail the mass matrix of the $SU(2)_L$ - doublet leptons of positive matter parity ($C' = +1$). These are the fields with the correct quantum numbers to play the role of the electroweak higgs doublets. Their mass matrix breaks up into two submatrices characterized by the S' -charge of the fields involved. The submatrix of the $S' = +1$ sector couples $H_k^{(2)} (k = 1, \dots, 4), \bar{H}_4^{(1)}$ and L_5 with $H_6^{(1)}, H_7^{(1)}, \bar{H}_1^{(2)}, \bar{H}_2^{(2)}$ and \bar{L}_3 . This is a 6×5 matrix with 5 eigenvalues $\sim M_X$. One linear combination, $h^{(2)}$, of the 6 fields, which have the correct quantum numbers to play the role of the higgs doublet giving mass to ordinary down-type quarks and leptons, remains naturally exactly massless. The corresponding $S' = -1$ submatrix couples $H_6^{(2)}, H_7^{(2)}, \bar{H}_1^{(1)}, \bar{H}_2^{(1)}$ and L_8 with $H_k^{(1)} (k = 1, \dots, 4), \bar{H}_4^{(2)}$ and \bar{L}_5 . This is a 5×6 matrix with 5 eigenvalues $\sim M_X$. Here again one linear combination, $h^{(1)}$, of the 6 fields, which have the correct quantum numbers to play the role of the higgs doublet giving mass to ordinary up-type quarks, remains naturally exactly massless. We see that the combined effect of the symmetries C' and S' solves in an elegant manner the gauge hierarchy problem. The mechanism takes into account all renormalizable as well as all the non-renormalizable superpotential terms.

The states which are not standard model singlets acquire masses $\sim M_X$ except the three ordinary light generations all of which have matter parity $C' = -1$. Two light lepton doublets as well as two charged light antilepton singlets have negative S' -charge while the third one has positive S' -charge. The same holds for the light quark doublets as well as the light up- type antiquarks. In contrast, there are two light down-type antiquarks with positive and only one with negative S' -charge. Taking into account the C' and S' -charges of the light higgs doublets $h^{(1)}, h^{(2)}$, we conclude that one light quark generation remains massless at tree-level.

As already emphasized, the fact that, below a unification scale M_X , we recover the MSSM is sufficient to guarantee the successful RG predictions for $\sin^2\theta_w$ and α_s . Choosing the values $M_s = 1\text{TeV}$ and $M_X = 1.3 \times 10^{16} \text{ GeV}$, a one-loop calculation gives the perfectly acceptable values $\sin^2\theta_w = 0.230$ and $\alpha_s = 0.116$ at the electroweak scale. Determining the scale M_c by the requirement that $\alpha_G(M_c) = 1$, we obtain the value $M_c = 2.4 \times 10^{18} \text{ GeV}$ which happens to be equal to $M_P/\sqrt{8\pi}$.

The only standard model singlets that are of interest to us are the negative matter parity ones with mass less than M_X . There are two such states with positive and three with negative S' - charge. They all have Majorana masses $\sim M_X^2/M_c$ from non-renormalizable superpotential couplings and play the role of the right-handed neutrinos in the see-saw mechanism. The order of magnitude of their mass allows for a solution of the solar neutrino puzzle through the MSW effect⁽⁴⁾ and at the same time does not exclude the possibility that the τ -neutrino contributes significantly to the missing mass of the universe. A sizable right-handed neutrino mass allows the implementation

of the mechanism of baryogenesis via leptogenesis.

By observing that the S' -charges of the electroweak higgs doublets $h^{(1)}$ and $h^{(2)}$ are opposite we conclude that the S' symmetry cannot remain unbroken. Actually, it must necessarily break by the expectation value of a standard model singlet in order for the necessary mixing between $h^{(1)}$ and $h^{(2)}$ and a mass term of the fermionic components of $h^{(1)}$ and $h^{(2)}$ to be generated. We assume that a gauge singlet field T with positive matter parity and negative S' -charge acquires a vacuum expectation value $\sim \frac{M_c}{M_X} M_s$ and a mass of the same order of magnitude. The necessary mixing $\sim M_s$ between $h^{(1)}$ and $h^{(2)}$ will then be generated by the non-renormalizable superpotential term $h^{(1)}h^{(2)}N_7T/M_c$. This term upon substitution of N_7 with its vacuum expectation value represents the dominant coupling between T and the MSSM states. Notice that the spontaneous breaking of the discrete symmetry S' does not lead to the well-known domain wall cosmological problem because of the $SU(3)_c$ -anomaly of the discrete symmetry $S'^{(5)}$.

One could obtain a more natural mechanism for generating a non-zero higgsino mass involving only scales $\sim M_X$ at the expense of introducing a Z_5 symmetry acting non-trivially only on gauge singlets. We introduce two such gauge singlets T and \bar{T} both with positive matter parity and negative S' -charge on which the new symmetry generator acts as $\alpha = \exp(2\pi i/5)$ and α^4 respectively. These singlets can acquire naturally expectation values $\sim M_X$ thereby breaking the S' symmetry at a superheavy scale. Remarkably enough, the higgsino mass generated by the non-renormalizable terms $h^{(1)}h^{(2)}N_7T^5/M_c^5$ and $h^{(1)}h^{(2)}N_7\bar{T}^5/M_c^5$ is still acceptably small whereas the singlets T and \bar{T} are obviously sufficiently decoupled. Discrete symmetries

broken spontaneously at superlarge scales $\sim M_X$ are not expected to give rise to domain wall formation in the inflationary universe scenario.

We conclude by summarizing our results. We presented a scheme in which a $G \equiv SU(3)^3$ supersymmetric gauge theory with one gauge coupling $\alpha_G = 1$ and symmetric spectrum among the three $SU(3)$ gauge groups arises as an effective 4-dimensional theory at a scale $M_c = M_P/\sqrt{8\pi}$. Spontaneous breaking of this gauge group at a scale $M_X = 1.3 \times 10^{16}\text{GeV}$ gives naturally rise exactly to the MSSM, a result which guarantees as a prediction the experimentally favored values of $\sin^2\theta_w$ and α_s . We relied on the combined effect of three Z_2 discrete symmetries, one to stabilize the proton, the second to act like a "matte parity" and the third to solve the gauge hierarchy problem. We easily obtain two neutrinos light enough to allow for a solution of the solar neutrino problem through the MSW effect and a third neutrino which possibly gives a considerable contribution to the "hot" component of the dark matter of the universe. The LSP is stable and could be the dominant "cold" component of dark matter. Lepton number asymmetry is generated through right-handed neutrino decays and is later partially transformed into baryon asymmetry by the non-perturbative baryon- and -lepton-number-violating effects of the standard electroweak theory. Finally, domain wall formation is avoided. We believe that models of this type should be seriously considered as viable alternatives to the usual minimal supersymmetric GUTs.

References

1. J. Ellis, S.Kelley and D.V.Nanopoulos, Phys.Lett.B249 (1990) 441;
U.Amaldi, W. de Boer and H. Furstenan, Phys. Lett. B260 (1991)
447; P.Langacker and M.X.Luo, Phys. Rev. D44 (1991) 817.
2. S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981)150; N. Sakai,
Z. Phys. C11 (1981) 153.
3. G. Lazarides, C. Panagiotakopoulos and Q. Shafi, Phys. Lett. B315
(1993) 325; E.B317 (1993)661.
4. L. Wolfenstein, Phys. Rev. D17 (1978) 2369; Phys. Rev. D20 (1979)
2634; S. P. Mikheyev and A. Yu. Smirnov, Usp. Fiz. Nauk. 153 (1987)
3.
5. J. Preskill, S.P. Trivedi, F. Wilczek and M.B. Wise, Caltec preprint
CALT-68-1718 (1991).